MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

ETM2126 – INFORMATION THEORY AND ERROR CONTROL CODING

(TE)

03 MAR 2020

9:00 A.M – 11:00 A.M.

(2 Hours)

INSTRUCTIONS TO STUDENT:

1. This exam paper consists of **5 pages** with **4 questions.**
2. Attempt **ALL questions**. All questions have equal weightage and the breakdown of marks within each question is given.
3. Please write all your answers in the answer booklet provided. Show all relevant steps to obtain maximum marks

Question 1

(a) Suppose the outcomes of rolling a dice have the following probabilities:

Table 1.1

Symbol	⚀	⚁	⚂	⚃	⚄	⚅
Probability	0.1	0.15	0.2	0.3	0.05	0.2

i. Find the average information per symbol.

[2 marks]

ii. If blocks of three symbols are transmitted at the rate of 1000 blocks per second, what is the average information rate for this source?

[3 marks]

(b) Construct the minimum variance Huffman code for the source with symbol probabilities given in Table 1.1. Show the details of your work using proper diagram.

[7 marks]

(c) Briefly describe or state the Shannon's lossless source coding theorem.

[4 marks]

(d) Suppose during the decoding process using the Lempel-Ziv algorithm, the search buffer's content is given in the figure below:

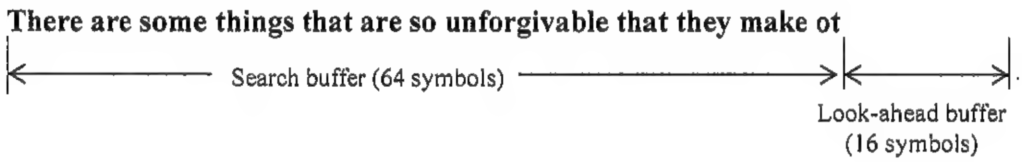

Figure 1.1

i. If the code consists of three parts: (Offset, Length, Next symbol), and ASCII code is used to represent the next symbol, what is the required size (in bits) for one code?

[3 marks]

ii. Complete the decoding process, if the following codes are received:

(63, 3, "space"), (53, 7, e), (18, 1, s), (8, 1, *l*), (25, 2, f), (46, 9, ".")

[6 marks]

Continued...

Question 2

(a) A discrete memoryless channel with input X and output Y is shown in Figure 2.1. The size of the input and output alphabets are 2 and 3 respectively.

Figure 2.1

Suppose $p(x_0) = 0.5$ and the channel matrix is given by

$$P = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 0.1 & 0.9 \end{bmatrix}$$

What is the amount of uncertainty (or average information) about the channel input that is *resolved* by observing the channel output?

[10 marks]

(b) Consider a binary symmetric channel (BSC) with transition probability $p = 0.01$. What is the maximum rate (in bits per channel use) at which the information can be transmitted over the channel with arbitrarily small amount of error?

[5 marks]

(c) Consider an additive white Gaussian noise (AWGN) channel that is bandwidth limited to B Hz and average received power constrained to P Watts. The power spectral density of the AWGN is given by $\frac{N_0}{2}$.

i. What is the maximum transmission capacity (in bits/s) that can be achieved for this channel? Give your answer in terms of B, P and N_0.

[3 marks]

ii. What happens to the information rate if the channel has no noise?

[3 marks]

iii. Show that as the bandwidth B tends to infinity, the channel capacity approaches $1.44\frac{P}{N_0}$. (Hint: $\ln(1 + x) \to x$, as $x \to 0$)

[4 marks]

Continued...

Question 3

(a) The generator matrix for a linear block code is given by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

i. Find the codeword for the input message 1101.

[3 marks]

ii. Calculate the code rate.

[2 marks]

iii. Find the parity-check matrix.

[2 marks]

iv. Suppose the binary sequence 1001111 is received and it contains one error bit. Construct the syndrome table and apply syndrome decoding to detect and correct the error.

[9 marks]

(b) Suppose the generator polynomial for a (7,4) cyclic code is given by $g(D)$.

i. If the message polynomial is represented as $m(D)$. Describe the steps to generate the *systematic* cyclic code.

[3 marks]

ii. In practice, systematic cyclic code can be generated using a linear shift register. If the generator polynomial is given by

$$g(D) = 1 + D + D^3$$

Draw the shift register for encoding the (7,4) cyclic code.

[3 marks]

iii. Suppose the received sequence is $y(D) = 1 + D^2 + D^3 + D^6$. Determine the syndrome polynomial.

[3 marks]

Continued...

Question 4

(a) Figure 4.1 shows a convolutional encoder with two input streams and three output streams:

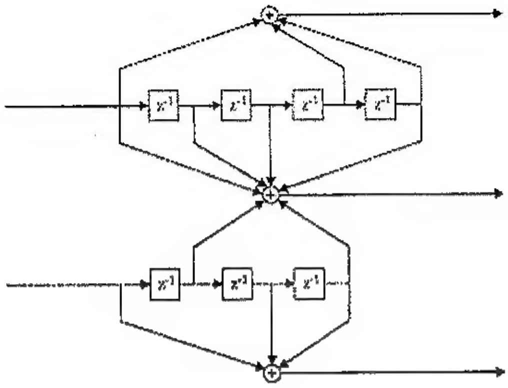

Figure 4.1

i. Find the code rate and number of possible states for this encoder.

[4 marks]

ii. What is the constraint length for the upper path of the encoder?

[2 marks]

iii. Suppose the generator polynomial from input stream i to the output stream j is denoted as $g_i^{(j)}$. Determine the set of generator polynomials that describe the encoder above.

[6 marks]

(b) The state transition table of a convolutional code is shown in Table 4.1

Table 4.1

Input Bit	Current State	Output	Next State
0	00	00	00
1	00	10	11
0	01	01	10
1	01	11	01
0	10	00	11
1	10	10	00
0	11	01	01
1	11	11	10

Apply Viterbi algorithm to decode the received sequence 0010110110. Draw the trellis diagram, trace the decisions on the diagram, and label the survivors' Hamming distance metrics at each node level. If a tie occurs in the metrics required for a decision, always choose the upper path.

[13 marks]

End of Paper